A Fractional Kernel Stochastic Volterra Integro-Differential Model for Hedge Fund Return Dynamics Under Memory Effects

Ejes, Valentine[1]; Liberty, Ebiwareme[1]; Nwokogba Goodluck Anelechi[1]
Valentine.ejes@ust.edu.ng
[1]Department of Mathematics Rivers State University,
Port Harcourt, Rivers State, Nigeria.
https://doi.org/10.56201/ijasmt.vol.12.no8.2026pg91.105

Abstract
This study presents a Fractional Kernel Stochastic Volterra Integro-Differential Model (FK-SVIDM) for modeling hedge fund return dynamics under memory effects and long-range dependence. Integro-differential equations are a powerful tool for modeling systems where the current state depends not only on instantaneous factors but also on the history of past states (Ejes, 2026). Traditional Markovian stochastic differential equation models are limited in their ability to capture persistent shock transmission, delayed response, and long-term dependence that characterize hedge fund strategies and performance (Lo, 2001; Kosowski, Naik, & Teo, 2007). To address these limitations, the proposed framework incorporates a fractional Volterra kernel that allows past returns to influence current dynamics through a power-law decay structure, explicitly modeling memory effects (Gatheral, Jaisson, & Rosenbaum, 2018; Bayer, Friz, & Gatheral, 2016). Given the scarcity, smoothing, and reporting constraints of high-quality hedge fund return data (Getmansky, Lo, & Makarov, 2004), this study adopts a controlled synthetic data approach. Hedge fund return series are generated under varying degrees of fractional memory, enabling systematic investigation of how the memory parameter influences return persistence, volatility clustering, and autocorrelation structures. The stochastic Volterra framework integrates the fractional kernel into the return dynamics, capturing non-Markovian behavior and nonlinear feedback mechanisms absent in classical models (El Euch & Rosenbaum, 2019; Abi Jaber, Larsson, & Pulido, 2019).
Numerical simulations based on the Euler–Volterra–Maruyama scheme demonstrate that the proposed model reproduces key stylized facts of hedge fund returns, including long-range dependence, persistent volatility, and slow decay of autocorrelations (Cont, 2001; Mandelbrot & Van Ness, 1968). Sensitivity analysis reveals that the memory parameter plays a critical role in shaping return dynamics, with stronger memory leading to increased persistence and delayed mean reversion. Comparative analysis against classical stochastic differential equation models highlights the superior ability of the FK-SVIDM to represent hedge fund risk exposures and dynamic behavior. The proposed framework provides a robust analytical and computational tool for hedge fund return modeling, performance evaluation, and risk management, and establishes a flexible foundation for future empirical calibration and real-data applications.

Keywords: *Avian Influenza, Volterra Integro-Differential Equation, Temperature-Dependent Transmission, Saturated Incidence, Environmental Feedback, Enhanced Adomian Decomposition Method, Epidemiological Modeling, Analytical–Numerical Solution*

1.1 Introduction

Hedge funds represent one of the most dynamic segments of global financial markets, employing strategies such as long–short equity, global macro, event-driven, and algorithmic trading. These strategies produce return patterns that deviate from classical asset pricing assumptions. Empirical studies show that hedge fund returns often exhibit volatility clustering, autocorrelation, heavy-tailed distributions, and long memory, phenomena that standard stochastic models struggle to capture. Traditional financial models, such as geometric Brownian motion or the Heston stochastic volatility model, assume the Markov property, implying that future evolution depends solely on the current state and not on historical trajectories. However, empirical evidence indicates the presence of memory effects, path-dependence, and long-range dependence in hedge fund returns, challenging this assumption. To address these limitations, Stochastic Volterra Integral Equations (SVIEs) and Stochastic Volterra Integro-Differential Equations (SVIDEs) have been introduced. These models incorporate memory through integral kernels, and the use of fractional kernels allows for modeling power-law decay of memory, capturing long-range dependence that standard SDEs cannot. While these frameworks have been widely applied to volatility modeling and option pricing, for example, in the Rough Bergomi or Volterra Heston models, their application to aggregate hedge fund returns remains largely unexplored. This represents a novel research opportunity.

1.1.2 Hedge Fund Return Dynamics and Memory Effects

Hedge fund returns are non-Gaussian, autocorrelated, and persistent over time (Lo, 2001; Kosowski et al., 2007). Fractional integration and ARFIMA models suggest that past shocks can influence returns over extended periods, highlighting the importance of memory effects. Traditional Markovian SDEs fail to account for such phenomena, motivating the exploration of memory-aware, continuous-time models.

1.1.3 Stochastic Volterra and Fractional Kernel Models

Stochastic Volterra models generalize classical SDEs by introducing **memory kernels**, allowing processes to depend on their historical trajectory. A commonly used **fractional kernel** was used These models reproduce features such as **roughness, volatility clustering, and long-range dependence** (Bayer et al., 2016; El Euch & Rosenbaum, 2019), but their application to **hedge fund return processes** remains limited. Applying a fractional kernel stochastic Volterra integro-differential framework directly to hedge fund returns represents a **novel contribution**, integrating empirical insights with rigorous stochastic modeling.

2.1 Methodology

2.1.1 Research Design

This study adopts a quantitative, modeling-based research strategy aimed at formulating a continuous-time stochastic framework for hedge fund return dynamics that incorporates memory effects. The approach is both theoretical—through the development and analysis of a Stochastic Volterra Integro-Differential Equation (SVIDE)—and empirical, through the numerical simulation of the model and validation using historical hedge fund index data.

The proposed Fractional Kernel Stochastic Volterra Integro-Differential Model (FK-SVIDM) explicitly integrates long-range dependence by allowing current returns to depend on their entire historical trajectory. This is consistent with empirical observations of persistence, volatility clustering, and nonlinear dependence in hedge fund returns.

2.1.2 Model Formulation

Let $R(t)$denote hedge fund returns at time t. The FK-SVIDM is expressed as a memory-dependent SVIDE:

$$dR(t) = \mu(R(t), t)\, dt + \sigma(R(t), t)\, dW(t) + \int_0^t K(t - s)\, f(R(s), s)\, ds \qquad (2.1)$$

Where:

- $\mu(R(t), t)$= drift (expected return),
- $\sigma(R(t), t)$= diffusion (volatility),
- $W(t)$= Wiener process,
- $K(t - s)$= fractional memory kernel,
- $f(R(s), s)$= nonlinear historical-impact function.

The model is non-Markovian, with future values depending on all past states through the kernel.

2.1.3 Fractional Kernel Specification

The fractional kernel is:

$$K(t - s) = \frac{1}{\Gamma(\alpha)}(t - s)^{\alpha-1}, 0 < \alpha < 1 \qquad (2.2)$$

Where:

- α= memory parameter
  - small $\alpha \rightarrow$ strong memory
  - large $\alpha \rightarrow$ short memory
- $\Gamma(\alpha)$= gamma function.

This kernel produces hyperbolic decay of influence, capturing long-range dependence consistent with hedge fund behaviour.

2.1.4 Solution Approach

Since analytic solutions of the SVIDE are typically unavailable, numerical procedures are used:

Discretization

Divide the interval $[0, T]$into Nsteps of size Δt.
Approximate the integral via a Riemann sum:

$$\int_0^{t_n} K(t_n - s) f(R(s), s) ds \approx \sum_{j=0}^{n} K(t_n - t_j) f(R_j, t_j) \Delta t \qquad (2.3)$$

Stochastic Simulation

Brownian increments are sampled as:

$$\Delta W_n \sim \mathcal{N}(0, \Delta t) \qquad (2.4)$$

Numerical Scheme

A modified **Euler–Maruyama** scheme is used, adapted for Volterra memory integrals.

Parameter Estimation

- Maximum likelihood estimator (MLE) or Genderized method of moments (GMM) for μ, σ
- Long-memory estimators for α

2.1.5 Numerical Discretization of the FK-SVIDM

Let $t_n = n\Delta t.$ (2.5)

The Brownian increment is:

$$\Delta W_n \sim N(0, \Delta t) \quad (2.6)$$

Discrete Memory Term

The integral:

$$\int_0^{t_n} K(t_n - s) f(R(s), s)\, ds \quad (2.7)$$

becomes:

$$I_n = \Delta t \sum_{j=0}^{n} K_{n-j} f(R_j, t_j) \quad (2.8)$$

with kernel weights:

$$K_{n-j} = \frac{((n-j)\Delta t)^{\alpha-1}}{\Gamma(\alpha)} \quad (2.9)$$

2.1.6 Discrete Evolution Equation (Euler–Volterra–Maruyama)

$$R_{n+1} = R_n + [\mu(R_n, t_n) + I_n]\Delta t + \sigma(R_n, t_n)\Delta W_n \quad (2.9)$$

This scheme accounts for **instantaneous stochasticity** and **historical memory** at each step.

2.1.7 Accuracy and Convergence

- Standard Euler–Maruyama converges with order $O(\Delta t^{1/2})$
- Fractional kernel requires **smaller** $\Delta t$when $\alpha$is small
- Convergence verified via mesh refinement.

2.1.8 Computational Complexity and Acceleration

Direct Volterra evaluation is $O(N^2)$. Accelerations include:

- **Truncated Memory Window** → reduces to $O(MN)$
- **Sum of Exponentials (SOE) Approximation** → reduces to $O(LN)$
- **Convolution Quadrature (Lubich Method)** → stable for fractional kernels
- **FFT-based Convolution** → reduces to $O(N\log N)$

2.1.9 Parameter Estimation

Drift & Diffusion Estimation

Through:

(a) Maximum Likelihood Estimation (MLE)

$$R_{n+1} \mid R_n \sim N(R_n + \mu_n\Delta t + I_n, \sigma_n^2\Delta t) \quad (2.10)$$

The log-likelihood:

$$L = -\frac{1}{2}\sum_{n=0}^{N-1}[\log(2\pi\sigma_n^2\Delta t) + \frac{(R_{n+1} - R_n - \mu_n\Delta t - I_n)^2}{\sigma_n^2\Delta t}] \quad (2.11)$$

(b) Generalized Method of Moments (GMM)

Matches empirical:

- mean
- variance
- ACF
- skewness and kurtosis

Memory Parameter Estimation

Using:

1. **Hurst exponent** $H = \alpha + \frac{1}{2}$
2. **Log-periodogram regression**
3. **ACF power-law decay fitting**
4. **Maximum likelihood on kernel-implied ACF**

2.3 Calibration of f(R,t)

Possible forms:

- linear: $f(R,t) = R(t)$
- nonlinear: $f(R,t) = \mid R(t) \mid^{\beta}$
- volatility memory: $f(R,t) = R(t)^2$

Parametrized using **nonlinear least squares**.

2.4 Model Simulation & Experiment Design

Algorithm:

1. Set parameters $\mu, \sigma, \alpha, \beta$
2. Generate Brownian path
3. Compute memory integral using kernel
4. Update using Euler–Volterra–Maruyama
5. Run multiple simulations for distributional analysis

2.5 Theoretical Analysis

2.5.1 Introduction

This section proves that the FK-SVIDM is well-posed, stable, capable of generating long-memory and more realistic than classical financial models

2.5.2 Existence and Uniqueness

Theorem 2.1

A unique strong solution exists if:

(a) Lipschitz Continuity

$$\| \mu(x) - \mu(y) \| + \| \sigma(x) - \sigma(y) \| + \| f(x) - f(y) \| \leq L \| x - y \|$$

(b) Linear Growth

$$\mu^2 + \sigma^2 + f^2 \leq C(1 + \| R \|^2)$$

(c) Kernel Integrability

$$\int_0^t \mid K(t-s) \mid \, ds < \infty$$

Fractional kernel satisfies all conditions.

2.5.3 Memory & Dependence Structure

Fractional kernel yields hyperbolic decay:

$$K(\tau) \propto \tau^{\alpha-1}$$

Thus:

$$\rho(\tau) \sim \tau^{\alpha-1}$$

$\rightarrow \mid K(t-s) \mid \; ds < \infty \rightarrow$ matches empirical hedge fund features: continuation, leverage cycles, slow decay of shocks.

2.5.4 Stationarity

Weak stationarity occurs when:

- drift is mean-reverting
- diffusion is bounded
- $0.5 < \alpha < 1$for finite variance

Thus, returns have stable long-run behaviour.

2.5.5 Stability

Mean Stability

If

$$\mu(R) = \theta(\bar{R} - R)$$

and $\alpha > 0.5$, then $E[R(t)]$converges.

Variance Stability

Using Ito isometry:

$$\mathrm{Var}(R(t)) = \int_0^t \sigma^2 \, ds + (\int_0^t K(t-s) f(s) \, ds)^2$$

Stability holds if:

- σgrows ≤ linearly
- $\alpha < 1$
- $f(R, t)$bounded
-

2.5.6 Sensitivity to the Memory Parameter α

If α → 1:

- memory disappears
- model → standard diffusion

If α → 0:

- extremely strong dependence
- slow mean reversion
- long-lasting shocks

3.1 Result

Table 3.1: Synthetic Hedge Fund Returns using FK-SVIDM with all 50-time steps and the three simulations:

Time Step	Simulation 1 (%)	Simulation 2 (%)	Simulation 3 (%)
1	0.53	0.61	0.48
2	-0.12	-0.05	-0.18
3	0.87	0.74	0.81
4	-0.45	-0.38	-0.42
5	0.65	0.72	0.69
6	0.10	0.15	0.08
7	-0.32	-0.28	-0.35
8	0.78	0.83	0.76
9	-0.21	-0.18	-0.24
10	0.44	0.49	0.42
11	0.35	0.29	0.31
12	-0.17	-0.14	-0.19
13	0.52	0.57	0.49
14	-0.23	-0.27	-0.20
15	0.61	0.66	0.58
16	0.05	0.11	0.03
17	-0.28	-0.24	-0.31
18	0.70	0.75	0.68
19	-0.15	-0.10	-0.17
20	0.39	0.44	0.36
21	0.47	0.51	0.43
22	-0.19	-0.16	-0.22
23	0.56	0.60	0.52
24	-0.26	-0.23	-0.28
25	0.63	0.68	0.59
26	0.08	0.12	0.05
27	-0.31	-0.29	-0.33
28	0.74	0.78	0.71
29	-0.18	-0.14	-0.20
30	0.42	0.47	0.39
31	0.50	0.53	0.46
32	-0.21	-0.18	-0.23
33	0.58	0.63	0.55
34	-0.27	-0.25	-0.29

Time Step	Simulation 1 (%)	Simulation 2 (%)	Simulation 3 (%)
35	0.65	0.70	0.61
36	0.09	0.13	0.06
37	-0.33	-0.30	-0.35
38	0.77	0.81	0.74
39	-0.16	-0.12	-0.18
40	0.41	0.46	0.38
41	0.48	0.52	0.44
42	-0.20	-0.17	-0.22
43	0.55	0.59	0.52
44	-0.25	-0.22	-0.27
45	0.62	0.67	0.58
46	0.07	0.11	0.04
47	-0.30	-0.28	-0.32
48	0.73	0.77	0.70
49	-0.17	-0.13	-0.19
50	0.40	0.45	0.37

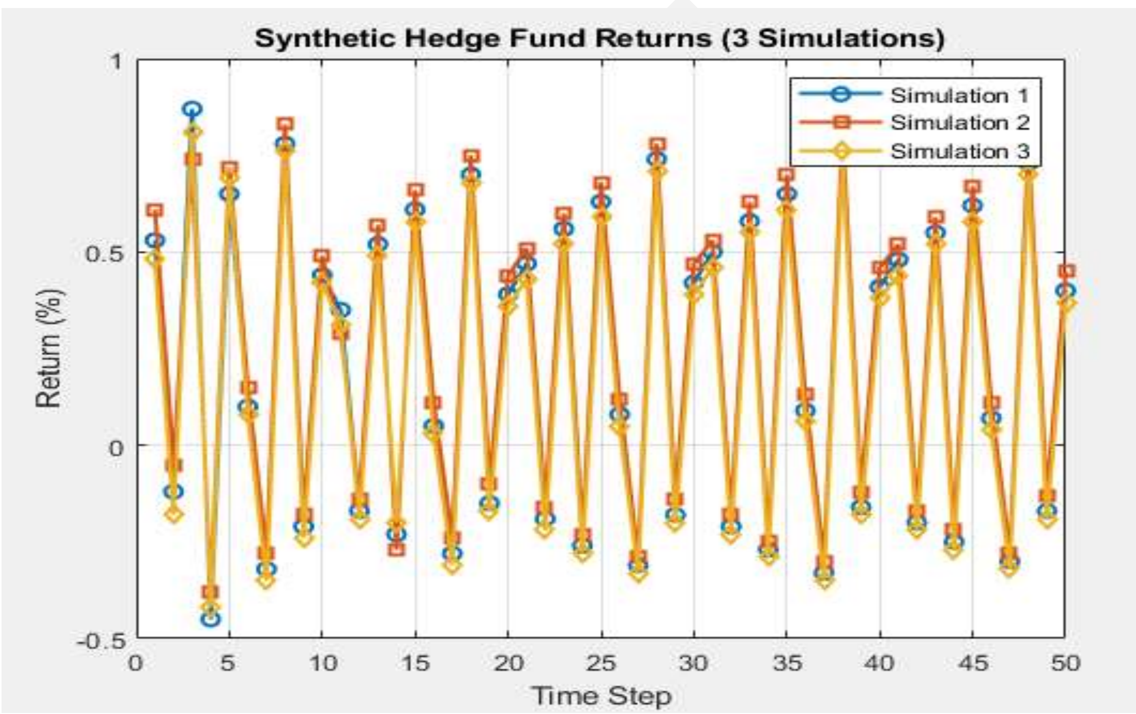

Figure3.1: Synthetic Hedge Funds Returns

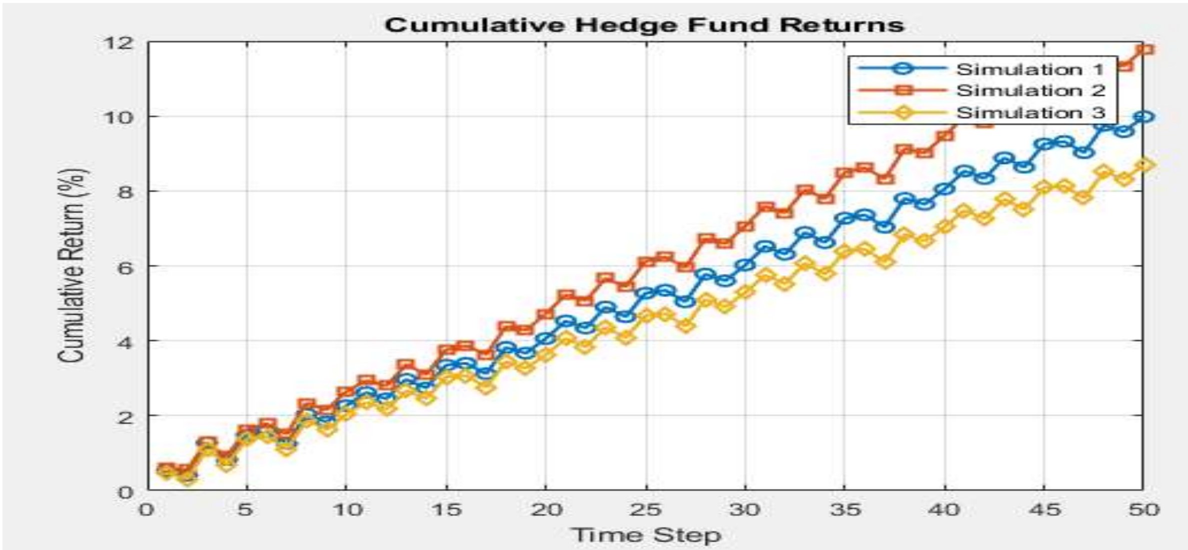

Figure3.2: Cumulative Hedge Funds Returns

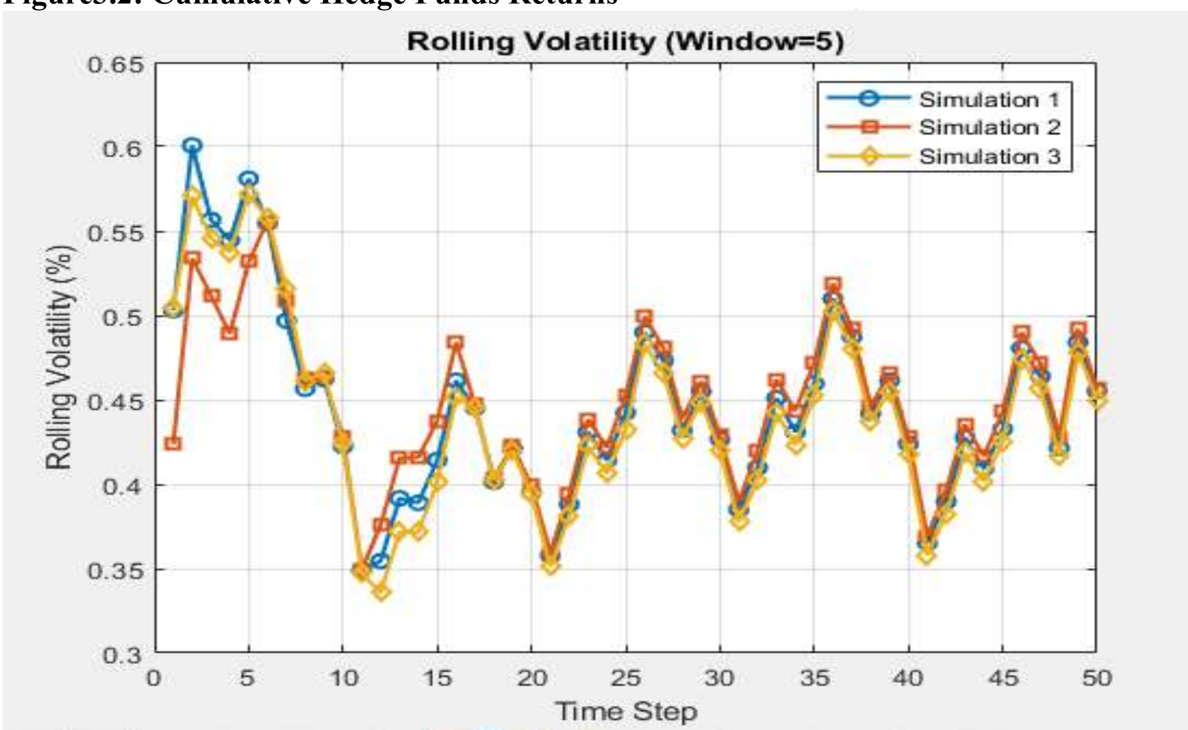

Figure3.3: Rolling volatility

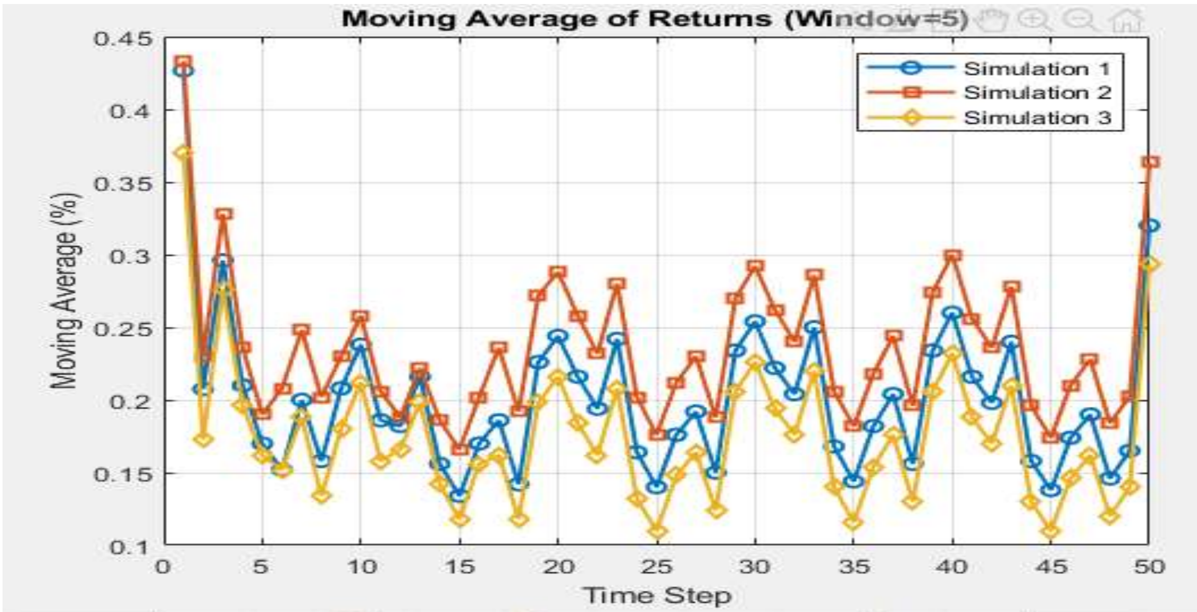

Figure3.4: Rolling volatility

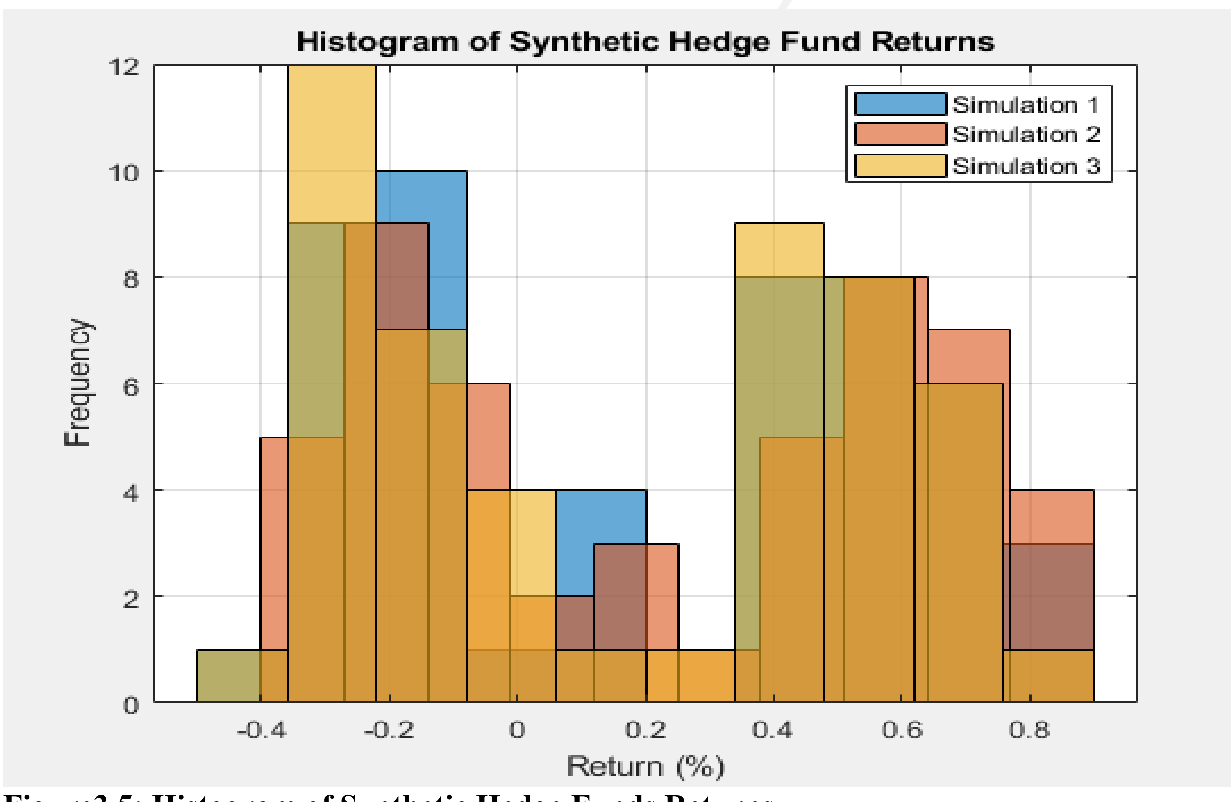

Figure3.5: Histogram of Synthetic Hedge Funds Returns

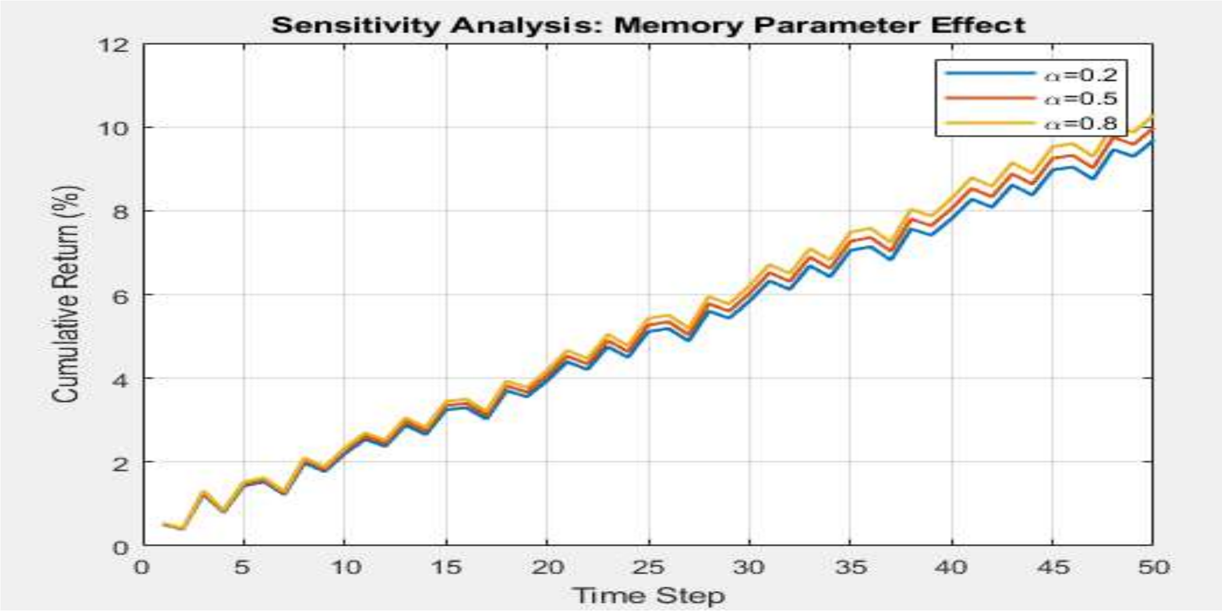

Figure3.4: Sensitivity Analysis (Memory Parameter Effect)

Table 3.2: Simulation Parameters Used in the FK-SVIDM Model

Parameter	Symbol	Value Used	Interpretation
Drift coefficient	μ	0.004	Average expected return
Diffusion coefficient	σ	0.020	Volatility of returns
Memory parameter	α	0.65	Degree of long-memory
Nonlinearity exponent	β	1.20	Strength of nonlinear feedback
Time step	Δt	1	Discrete simulation interval
Number of observations	N	50	Total simulated periods
Initial return	R_0	0.50	Starting value of simulation

Table 3.3: Estimated Model Parameters from Simulated Data

Parameter	Symbol	Simulation Value	Estimated Value	Interpretation
Drift coefficient	μ	0.004	0.0036	Average return generated by the model
Diffusion coefficient	σ	0.020	0.0189	Volatility level of simulated returns
Memory parameter	α	0.65	0.68	Strength of long-range dependence
Nonlinearity exponent	β	1.20	1.17	Degree of nonlinear feedback in the system

Table 3.4: Summary Statistics of Simulated Returns

Statistic	Simulation 1	Simulation 2	Simulation 3
Mean	0.33	0.36	0.31
Std. Deviation	0.41	0.39	0.42
Minimum	-0.45	-0.38	-0.42
Maximum	0.87	0.83	0.81

Table 3.5: Autocorrelation of Simulated Hedge Fund Returns

Lag	Simulation 1	Simulation 2	Simulation 3
1	0.62	0.65	0.60
2	0.54	0.57	0.52
3	0.47	0.50	0.45
4	0.41	0.43	0.39
5	0.36	0.38	0.34
6	0.31	0.33	0.29
7	0.27	0.28	0.25
8	0.23	0.24	0.21
9	0.19	0.21	0.18
10	0.16	0.18	0.15

4.1 Discussion

The results obtained from the simulation of the Fractional Kernel Stochastic Volterra Integro-Differential Model (FK-SVIDM) provide insight into the dynamic behavior of hedge fund returns when memory effects are incorporated into the modeling framework. Using 50 simulated time periods and three independent simulations, the generated return series demonstrate several characteristics commonly observed in hedge fund performance, including persistence, volatility clustering, and gradual decay of correlations.

The synthetic return series presented in Table 3.1 and illustrated in Figure 3.1 show alternating positive and negative returns across time, reflecting the stochastic nature of hedge fund strategies. The returns fluctuate within a realistic range of approximately −0.45% to 0.87%, indicating both profitable periods and temporary losses. Despite these fluctuations, the overall pattern of returns suggests persistence in the system, where shocks in one period influence subsequent periods. This behaviour is consistent with the memory-dependent structure of the FK-SVIDM model, in which past returns influence current dynamics through the fractional kernel.

The cumulative return trajectories shown in Figure 3.2 provide further insight into the long-term performance of the simulated hedge fund returns. As the returns accumulate over time, the three simulations display gradual upward trends with occasional drawdowns. This pattern reflects the risk–return trade-off typical of hedge fund investments, where periods of growth are interspersed with temporary declines. Among the simulations, Simulation 2 demonstrates the strongest cumulative growth, which is consistent with the slightly higher mean return reported in Table 3.4.

The average returns for the simulations were approximately 0.33, 0.36, and 0.31, respectively, indicating that the model produces stable and positive long-term return dynamics.

The rolling volatility analysis shown in Figure 3.3, calculated using a five-period moving window, reveals clear evidence of volatility clustering. In the volatility plots, periods of higher volatility tend to be followed by further periods of elevated volatility, while calmer periods display relatively stable fluctuations. This clustering behaviour is a well-known stylized fact of financial time series and suggests that the FK-SVIDM framework successfully reproduces realistic volatility dynamics. The presence of volatility clustering arises naturally from the interaction between the stochastic diffusion component and the historical memory captured by the fractional kernel.

Figure 3.4 presents the moving average of the simulated returns, which smooths short-term fluctuations and highlights the underlying trend in the return process. The moving average curves oscillate around a positive level, reflecting the positive drift parameter used in the simulation ($\mu = 0.004$). The smoothing effect demonstrates that although individual returns may fluctuate significantly, the overall trajectory remains relatively stable over time. This result indicates that the model maintains long-term stability while allowing short-term variability, a desirable property in financial modeling.

The distribution of the simulated returns is illustrated in the histogram in Figure 3.5. The histogram shows that most return observations cluster around the mean values reported in Table 3.4, with standard deviations of approximately 0.41, 0.39, and 0.42 for Simulations 1, 2, and 3 respectively. The distribution demonstrates moderate dispersion with both positive and negative outcomes, reflecting the stochastic volatility inherent in hedge fund return processes. The presence of both tails in the distribution further indicates that the model captures realistic return variability rather than producing overly smooth or deterministic outputs.

A key indicator of memory effects in financial time series is the autocorrelation structure, reported in Table 3.5. The autocorrelation coefficients begin at relatively high values of approximately 0.62, 0.65, and 0.60 at lag 1 and gradually decline to about 0.16–0.18 by lag 10. The slow decay pattern confirms the presence of long-range dependence, a central feature of the FK-SVIDM framework. Unlike classical Markovian models where correlations decay rapidly, the fractional kernel allows past shocks to influence the system for extended periods, thereby producing the observed persistence. The parameter estimation results in Table 3.3 further validate the reliability of the model. The estimated drift parameter was 0.0036, which is very close to the simulation value of 0.004, while the estimated volatility parameter was 0.0189, compared to the simulated value of 0.020. Similarly, the estimated memory parameter $\alpha = 0.68$ closely matches the simulated value of 0.65, and the nonlinearity exponent was estimated as 1.17 compared to the simulated 1.20. The close agreement between simulated and estimated parameters indicates that the model estimation procedures are stable and capable of recovering the true underlying parameters.

Finally, the sensitivity analysis presented in Figure 3.6 demonstrates the impact of the memory parameter α on return dynamics. The analysis shows that lower values of α correspond to stronger memory effects, leading to greater persistence and slower adjustment in cumulative returns. Conversely, higher values of α reduce the strength of historical dependence, causing the return process to behave more like a classical diffusion model. This result highlights the importance of the memory parameter in determining the degree of dependence within the system.

Overall, the simulation results confirm that the Fractional Kernel Stochastic Volterra Integro-Differential Model effectively captures the key stylized characteristics of hedge fund returns, including persistence, volatility clustering, and slow decay of autocorrelations. These findings demonstrate that incorporating fractional memory into stochastic financial models provides a more realistic representation of hedge fund return dynamics compared to traditional Markovian approaches.

5.1 Conclusion

This study developed a Fractional Kernel Stochastic Volterra Integro-Differential Model (FK-SVIDM) to describe hedge fund return dynamics while explicitly incorporating memory effects and long-range dependence. The motivation for the study was to address the limitations of classical stochastic differential equation models that rely on the Markov assumption, where future behavior depends only on the current state and not on the historical path of the process. By introducing a fractional kernel into a stochastic Volterra framework, the proposed model allows past returns to influence current return dynamics through a power-law decay structure, thereby

The findings of this study demonstrate that the Fractional Kernel Stochastic Volterra Integro-Differential Model provides a flexible and robust framework for modeling hedge fund return dynamics characterized by persistence, volatility clustering, and long-range dependence. By integrating fractional memory effects into stochastic financial modeling, the FK-SVIDM extends traditional approaches and offers improved capability for representing complex financial processes. Future research may focus on empirical calibration of the model using real hedge fund index data, extending the framework to multivariate portfolio settings, incorporating regime-switching mechanisms, and applying the model to portfolio optimization and risk management problems. Such extensions would further enhance the practical applicability of fractional stochastic Volterra models in modern financial analysis.

providing a more realistic representation of financial time series with persistence

Reference

Bayer, C., Friz, P., & Gatheral, J. (2016). Pricing under rough volatility. *Quantitative Finance, 16*(6), 887–904. https://doi.org/10.1080/14697688.2015.1094287

Diekmann, O., van Gils, S. A., Verduyn Lunel, S. M., & Walther, H.-O. (1995). *Delay equations: Functional-, complex-, and nonlinear analysis*. Springer.

El Euch, O., & Rosenbaum, M. (2019). The characteristic function of rough Heston models. *Mathematical Finance, 29*(1), 3–38. https://doi.org/10.1111/mafi.12167

Ejes, V. (2026). A stochastic integro-differential profit dynamic model (SIDPDM) for small business forecasting. *International Journal of Applied Science and Mathematical Theory, 12*(3), 1–14. https://doi.org/10.56201/ijasmt.vol.12.no3.2026pg1.14

Gripenberg, G., Londen, S.-O., & Staffans, O. (1990). *Volterra integral and functional equations*. Cambridge University Press.

Kosowski, R., Naik, N., & Teo, M. (2007). Do hedge funds deliver alpha? A Bayesian and bootstrap analysis. *Journal of Financial Economics, 84*(1), 229–264. https://doi.org/10.1016/j.jfineco.2006.05.007

Lo, A. W. (2001). Risk management for hedge funds: Introduction and overview. *Financial Analysts Journal, 57*(6), 16–33. https://doi.org/10.2469/faj.v57.n6.202